Secure Certification of Mixed Quantum States

Frédéric Dupuis, Serge Fehr, **Philippe Lamontagne** and Louis Salvail

Quantum state certification

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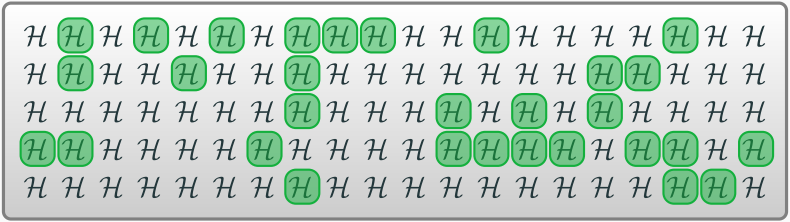

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- Measure $\mathcal{H}$ with $\{|\psi\rangle\langle\psi|, \mathbb{I} - |\psi\rangle\langle\psi|\}$
- If result is $|\psi\rangle$ for every $\mathcal{H}$, then *most* of the remaining positions are in state $|\psi\rangle$ with overwhelming probability [BF10].
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No local measurement for a discrete notion of errors for mixed states

A mixed state certification protocol

Possible to verify that a qubit is in state φ if we have access to its purifying register.

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Verifier wants to certify that his state is close to $\varphi^{\otimes n}$.

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V. Choose a random sample, announce it to prover.

P. Send R for each position in sample.

V. Measure $\{|\varphi\rangle\langle\varphi|_{AR}, \mathbb{I} - |\varphi\rangle\langle\varphi|_{AR}\}$ for each joint system AR in sample.

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Interaction is necessary

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$$\left(\frac{|0\rangle\langle 0|}{2}+\frac{|1\rangle\langle 1|}{2}\right)^{\otimes n} \quad \text{from} \quad \overbrace{|0\rangle|0\rangle\ldots|0\rangle}^{\approx n/2 \text{ times}}\overbrace{|1\rangle|1\rangle\ldots|1\rangle}^{\approx n/2 \text{ times}}$$

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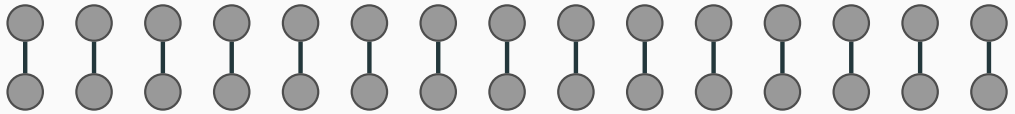

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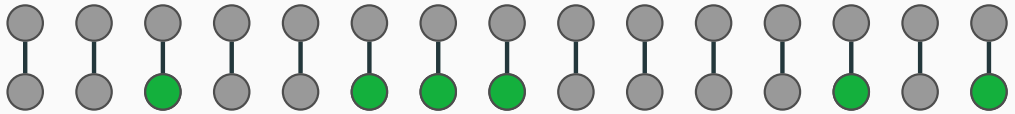

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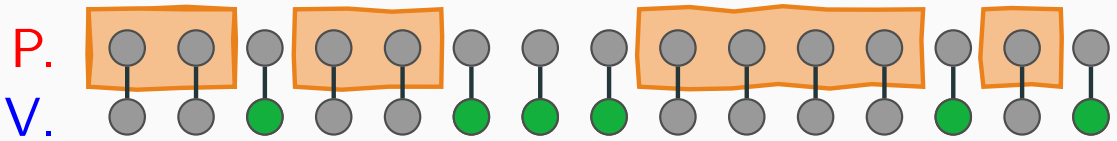

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2. Measures qubits

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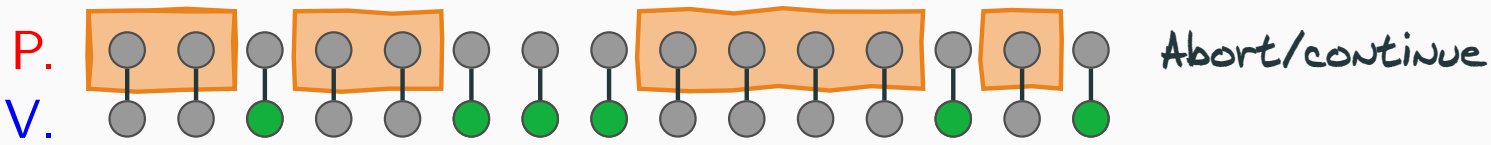

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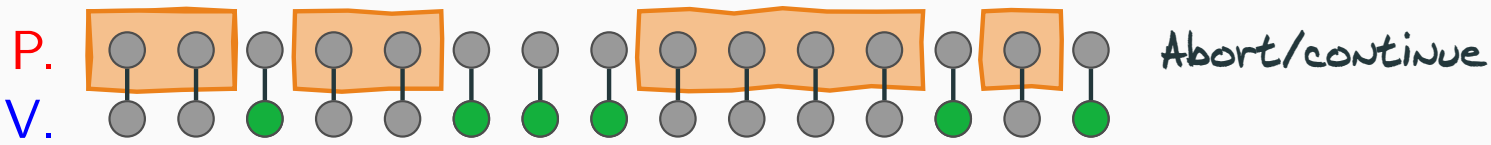

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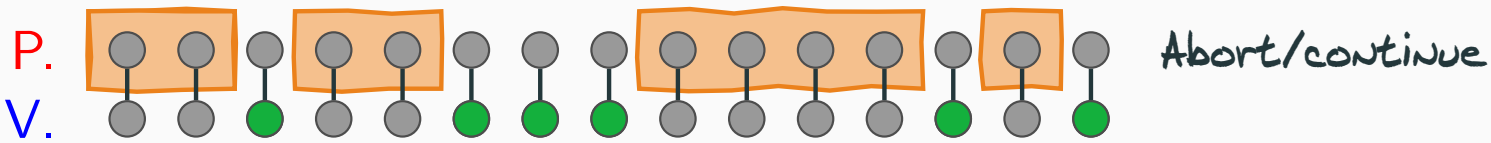

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Post-selection

Example

Prepare $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)^{\otimes n}$, measure positions outside of sample, abort if result $\neq |0\rangle^{\otimes n-k}$.

Resulting state always $|0\rangle^{\otimes n-k}$

What *can* the prover do ?

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An "undetectable" attack

The prover can

- prepare the honest state, up to a few errors,
- prepare a mixture/superposition of such states,
- purify this mixture, and
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The mixed state certification Theorem

Main Result

For any strategy of the prover, if the verifier accepts, his output state ρ_{A^n} satisfies

$$\rho_{A^n} \leq p_n \cdot \psi_{A^n} + \sigma$$

where p_n is a fixed-degree polynomial in n, ψ_{A^n} is the reduced operator of an ideal state $|\psi\rangle_{A^n R^n E}$ and $\mathrm{tr}(\sigma) \leq \mathrm{negl}(n)$.

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Application to Cryptography

For any POVM operator E^{bad} of a "bad" outcome,

$$\mathrm{tr}\left(E^{bad}\rho_{A^n}\right) \leq p_n \cdot \mathrm{tr}\left(E^{bad}\psi_{A^n}\right) + \mathrm{negl}(n)$$

Bad outcome on real state has negligible probability if $\mathrm{tr}(E^{bad}\psi_{A^n})$ is negligible.

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Generalisations and special cases

Sufficient conditions

Invariance under permutations. *Equivalent* to protocol where verifier permutes his registers with random π and announces π to the prover.

Behaves well on "easy" state. The verifier detects any cheating attempt with overwhelming probability on a state of the form $\sigma^{\otimes n}$ for σ *distant* from reference state φ.

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Corollary

Theorem implies security of

- *a local measurement certification protocol for* $\varphi = \frac{\mathbb{I}}{2}$,
- *pure state certification [BF10], and*
- *a "distributed" pure state certification protocol [DDN14] not covered by [BF10].*

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Behaves well on "easy" state. The verifier detects any cheating attempt with overwhelming probability on a state of the form $\sigma^{\otimes n}$ for σ *distant* from reference state φ.

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Theorem implies security of

- *a local measurement certification protocol for* $\varphi = \frac{\mathbb{I}}{2}$,
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Application : secure two-party randomness generation

Secure Two-Party Randomness Generation

Goal

Produce $X_A, X_B \in \{0,1\}^n$ such that

- $X_A = X_B$ if Alice and Bob are both honest,
- $\mathrm{H}_\infty(X_A) \geq (1-\epsilon)n$ and $\mathrm{H}_\infty(X_B) \geq (1-\epsilon)n$ except with negligible probability.

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- Alice prepares $|\Psi\rangle_{AB}^{\otimes N}$ and sends B^N to Bob.
- Bob *certifies* that most of his registers are close to $\frac{\mathbb{I}}{2}$.
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Our main result ensures that the measurement outcome will have near maximal min-entropy

Thank you !


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